Instituto Superior de Economia e Gestão

Masters in Economics and Masters in Monetary and Financial Economics

Microeconomics Exam

Maximum duration: 2h30m **15-1-2014**

Answer any four questions (each question carries 5 marks)

Question 1

A consumer's preferences can be described by the utility function $u(x_1,x_2) = \ln x_1 + 2\ln x_2$.

a) Find the consumer's Marshallian demand functions. (2 marks)

Marshallian demand gives the quantities that maximise utility subject to the budget constraint. This means that the following conditions must hold:

$p_1x_1 + p_2x_2 = y$ (1)

$MRS = (\partial u(x_1,x_2)/\partial x_1)/(\partial u(x_1,x_2)/\partial x_2) = p_1/p_2$

$\Leftrightarrow (1/x_1)/(2/x_2) = p_1/p_2 \Leftrightarrow x_2/(2x_1) = p_1/p_2 \Leftrightarrow x_2 = 2x_1p_1/p_2$ (2)

Substituting (2) into (1) yields the Marshallian demand function for good 1:

$p_1x_1 + p_2 2x_1p_1/p_2 = y \Leftrightarrow 3p_1x_1 = y \Leftrightarrow x_1 = y/(3p_1)$ (3)

Substituting (3) into (2) yields the Marshallian demand function for good 2: $x_2 = 2y/(3p_1)$

b) Verify that the demand functions you found satisfy the Engel aggregation, that is, $s_1\eta_1 + s_2\eta_2 = 1$ where $s_i = p_ix_i/y$ (the share income spent on good i) and η_i is the income-elasticity of demand of good i. (1 mark)

$\eta_1 = \partial x_1(p_1,y)/\partial y \times (y/ x_1) = 1/(3p_1) \times (y/(y/(3p_1)) = 1.$

$\eta_2 = \partial x_2(p_2,y)/\partial y \times (y/ x_2) = 2/(3p_1) \times (y/(2y/(3p_1)) = 1.$

Then $s_1\eta_1 + s_2\eta_2 = s_1 \times 1 + s_2 \times 1 = 1$ because $s_1 + s_2 = 1$.

Anyway $s_1 = p_1x_1/y = p_1(y/(3p_1))/y = 1/3$, and $s_2 = p_2x_2/y = p_2(2y/(3p_2))/y = 2/3$, so:

$s_1\eta_1 + s_2\eta_2 = (1/3) \times 1 + (2/3) \times 1 = 1.$

c) Show that (regardless of the utility function) if the consumer spends all his income (the usual assumption) then the Engel aggregation always holds. (2 marks)

See Jehle and Reny p. 61.

Question 2

a) An expected-utility maximiser has an utility function $u(w) = \ln w$, where w is wealth and ln is the natural logarithm. The agent has initial wealth w and can invest part of it only in a particular risky asset. This asset will have a rate of return of 100% with probability 50% and a rate of return of 0% also with 50% probability. What percentage of his wealth will the agent wish to invest in this asset? (2.5 marks)

The capital invested will double with probability 50% and stay the same with probability 50%, whereas the wealth not invested will stay the same. Then, assuming the agent cannot borrow, he will invest his whole wealth. [The question was not meant to be so easy!]

b) A risk-averse expected-utility maximiser has initial wealth w and can invest part of it only in an asset that has a rate of return a with probability p, and a (negative) rate of return $-b$ with probability $1 - p$. Show that the agent will choose to invest nothing in this asset if and only if the asset has zero or negative expected rate of return. (2.5 marks)

Let A be the capital invested in the risky asset. His final wealth will then be $w + aA$ with probability p and $w - bA$ with probability $1 - p$. His expected utility is then:

$EU(A) = pu(w + aA) + (1 - p)u(w - bA)$ (1)

$EU'(A) = pau'(w + aA) - (1 - p)bu'(w - bA)$ (2)

The rate of return of the risky asset is $pa - (1 - p)b$. It is negative or zero if if $pa \leq (1 - p)b$ (3)

As the agent is risk-averse $u'(.)$ is decreasing; then for any $A > 0$ $w - bA < w + aA$, which means that

$u'(w + aA) < u'(w - bA)$ (4)

Combining (3) and (4) implies $pau'(w + aA) < (1 - p)bu'(w - bA)$, so $EU'(A)$, expression (2), is negative for any $A > 0$. This means that decreasing A increases expected utility. Then a risk-averse agent will invest nothing if the rate of return is negative or zero. It remains to show that he will invest nothing *only* if the rate of return is negative or zero, that is, he will invest something if the rate of return is positive. He will always invest something if $EU'(0) > 0$ (meaning that increasing A from zero to something above zero increases expected utility):

$EU'(0) = pau'(w) - (1 - p)bu'(w) = (pa - (1 - p)b)\, u'(w)$ (5)

$EU'(0) > 0$ if the expected return, $pa - (1 - p)b)$, is positive, because $u'(w) > 0$. So if the rate of return is positive he will invest something.

Question 3

Briefly explain what is preference reversal, its several interpretations and relevance for economic theory, and the empirical support for the different interpretations. (5 marks)

Answer outline: 1) What preference reversal (PR) is (first pages of Braga 2004); 2) Interpretations (and relevance) – (a) violation of procedure invariance (PR rather damaging for standard economic theory of preference), (b) intransitive preferences and (c) preferences that do not obey the independence axiom of expected utility (awkward, but not as damaging as (a)), and (d) subjects' inexperience (PR laboratory phenomenon with little relevance for decision making in the real world); 3) Empirical support for the interpretations – Experiment by Kahneman et al 1990, reported in Braga 2004, casts doubt on (b) and (c) but not on (a); Cox and Grether 1996, reported in Braga 2004, lends some support to (d).

Question 4

Consider a competitive industry composed of 48 identical firms. Firms produce according to the Cobb-Douglas technology $q = x^{0.5}k^{0.5}$, where x is labor, a variable input, and k is plant size, which is fixed in the short-run. Assume that each firm operates a plant size $k = 1$, that the price of labor is $w_x = 4$, and that the price of capital is $w_k = 1$.

a) Compute a firm's profit function in the short run as a function of p, the price of output. (1.5 marks)

$$\pi = \frac{p^2}{16} - 1$$

b) Determine the market supply function. (1 mark)

The individual supply function is $q^S = \frac{p}{8}$ and market supply is $Q^S = 48 * \frac{p}{8} = 6p$.

Now assume that market demand is given by $Q^d(p) = 294/p$.

c) Determine the short run equilibrium price, output per firm, and firm profit. (1.5 marks)

$$p = 7, q = 42, \pi = \frac{33}{16}.$$

d) the values determined in c) be part of a long run equilibrium? Explain. (1 mark)

No. If one additional firm enters the market, it will have positive profits. Therefore, this is not a long run equilibrium.

Question 5

Consider the game in extensive form represented in Figure 1.

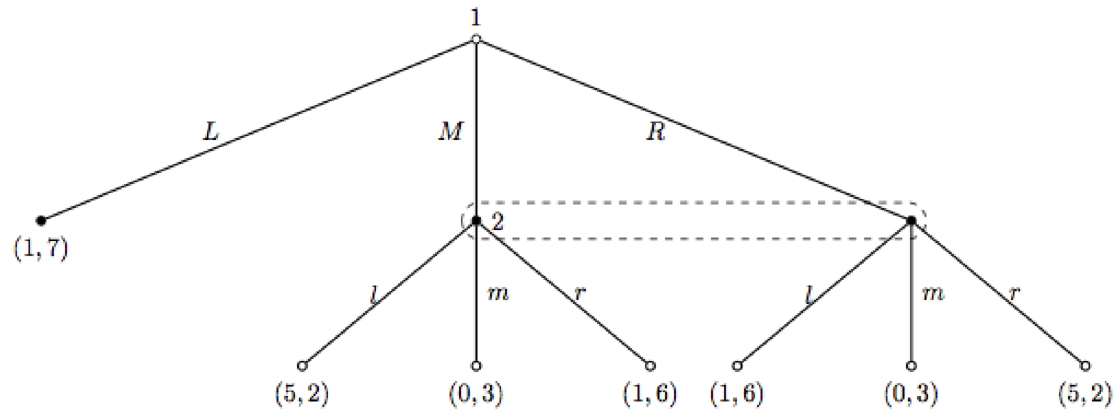

Figure 1

a) Write the game in normal form. (0.5 marks).

$G=(I,(S_i)_{i=1,2}, (u_i)_{i=1,2})$, $I=\{1,2\}$, $S_1=\{L,M,R\}$, $S_2=\{l,m,r\}$ and u_i, i=1,2, is in the following table:

	l	m	r
L	1,7	1,7	1,7
M	5,2	0,3	1,6
R	1,6	0,3	5,2

b) Compute the Nash equilibria of this game. (1.5 marks)

The unique Nash equilibrium in pure strategies is (L,m).

c) How many subgames does this game have? (0.5 marks)

One, the game itself.

d) Determine the subgame perfect Nash equilibria in pure strategies. (1 mark)

SPE={(L,m)}

e) Check whether the equilibria found in d. is sequential. (1.5 marks)

No, when given the chance to play, player 2 will not chose m, for any beliefs he may have. In fact, a combination of l and r, e.g., 50% in l and 50% in r, delivers higher expected payoff (0.5*2+0.5*6=4) than m (which always pays 3).

Question 6

Consider a principal-agent problem where there are two possible results: x_1 = 25000 and x_2 = 50000 . The agent can chose between two levels of effort e_1 and e_2. The probability of obtaining each result depends on effort according to the following table:

	x_1	x_2
e_1	0.25	0.75
e_2	0.5	0.5

(For example, the probability of obtaining x_1 with e_1 is 0.25.)
The principal is risk-neutral and the agent is risk averse. The principal's utility function is $v = x - w$. The agent's utility function is: $u(w,e) = \sqrt{w} - g(e)$ with $g(e_1) = 40$ and $g(e_2) = 20$. The reservation utility of the agent is $\overline{U} = 120.$

a) Assuming that information is symmetric (i.e., effort is observable), determine the optimal contract for each level of effort. What is the optimal level of effort for the principal? (1.5 marks)

From the individual rational constraints, one obtains *w*= 19600 to induce e_2 and *w*= 25600 to induce $e_1.$ Therefore, e_2 leads to an expected profit of 17900, whereas e_1 leads to an expected profit of 18150. The principal prefers $e_1.$

b) Now assume that effort is not observable (whereas *x* is verifiable). Determine the optimal contract to induce each level of effort. What is the optimal level of effort for the principal? (2.5 marks)

To induce e_2, the wage is constant and equal to *w*= 19600. To induce $e_1,$ the wage has to depend on *x*. Using individual rationality and incentive compatibility constraints, we obtain: If $x = x_1,$ w=10000; if $x = x_2,$ w=32400. Therefore, e_2 leads to a profit of 17900, whereas e_1 leads to an expected profit of 16950. The principal prefers $e_2.$

c) How does moral hazard influence the optimal contract? What is the efficiency loss associated to the moral hazard problem? (1 mark)

In this example, moral hazard leads to a change in the optimal level of effort.

With symmetric information, expected profit is 18150 and the agent's utility is 120. With moral hazard, expected profit is 17900 and the agent's utility is still the reservation level 120.

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Microeconomics Exam

Duração máxima: 2h30m **15-1-2014**

Responda a quaisquer quarto questões (cada questão vale cinco valores)

Questão 1

As preferências dum consumidor podem ser descritas pela função de utilidade $u(x_1,x_2) = \ln x_1 + 2\ln x_2$.

a) Obtenha a funções de procura Marshalianas deste consumidor. (2 valores)

b) Verifique que as funções de procura obtidas respeitam a agregação de Engel, isto é, $s_1\eta_1+s_2\eta_2=1$ onde $s_i= p_ix_i/y$ (a proporção do rendimento gasta no bem i) e η_i é a elasticidade-rendimento da procura do bem i. (1 valor)

c) Mostre que (independentemente da função de utilidade) se o consumidor gastar todo o seu rendimento (a premissa habitual), então a agregação de Engel é sempre respeitada. (2 valores)

Questão 2

a) Um maximizador da utilidade esperada tem a função de utilidade $u(w) = \ln w$, onde w é a riqueza e ln é o logarítmo natural. O agente tem uma riqueza inicial w e pode investor parte dela apenas num certo ativo com risco. Este ativo tem uma taxa de rendimento de 100% com 50% de probabilidade e de 0% também com 50% de probabilidade. Que percentagem da sua riqueza vai o agente investir neste ativo? (2,5 valores)

b) Um maximizador da utilidade esperada avesso ao risco tem uma riqueza inicial w e pode investor parte dela apenas num ativo que tem uma taxa de rendimento a com probabilidade p e uma taxa de rendimento (negativa) $-b$ com probabilidade $1 - p$. Mostre que o agente vai investir nada neste ativo se e só se o ativo tiver uma taxa de rendimento esperada nula ou negativa. (2,5 valores)

Question 3

Explique sucintamente a inversão de preferências, as várias intrepertações deste fenómeno e respectiva relevância para a teoria económica, e o apoio empírico às diferentes interpretações. (5 valores)

Questão 4

Considere um mercado em concorrência perfeita onde existem 48 empresas idênticas. Cada empresa utiliza uma tecnologia de produção do tipo Cobb-Douglas $q = x^{0,5}k^{0,5}$, onde q representa a produção de uma empresa, x é a quantidade do factor de produção variável trabalho e k representa o tamanho da fábrica, que é fixo no curto prazo. Suponha ainda que cada empresa opera com $k = 1$, que o salário é $w_x = 4$ e que o preço do capital é $w_k = 1$.

a) Obtenha a função lucro de curto prazo de cada empresa (como função de p, o preço do produto). (1.5 valores)

b) Determine the market supply function. (1 valor)

Assuma agora que a procura de mercado é $Q^d(p) = 294/p$.

c) Obtenha o preço, a quantidade produzida por cada empresa e o lucro no equilíbrio de curto prazo. (1.5 valores)

d) Será que os valores determinados em c. podem ser parte de um equilíbrio de longo prazo? Explique. (1 valor)

Questão 5

Considere o jogo na forma extensiva representado na Figura 1.

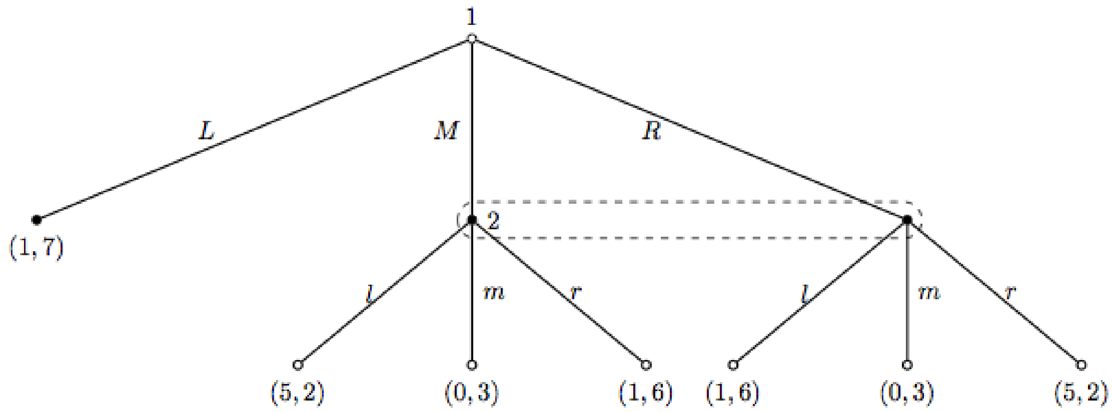

Figure 2

a) Escreva o jogo na forma normal. (0.5 valores)

b) Calcule os equilíbrios de Nash equilibria deste jogo. (1.5 valores)

c) Quantos subjogos tem este jogo? (0.5 valores)

d) Determine os equilíbrios de Nash perfeitos nos subjogos em estratégias puras. (1 valor)

e) Verifique se o(s) equilíbrio(s) encontrado(s) em d. é (são) equilíbrios sequenciais. (1.5 valores)

Questão 6

Considere a relação entre um delegante e um agente, em que existem apenas dois possíveis resultados x_1 = 25000 e x_2 = 50000 . O agente pode escolher entre dois níveis de esforço $\mathbf{e_1}$ e $\mathbf{e_2}$. A probabilidade de cada um dos resultados depende do nível de esforço, de acordo com a tabela seguinte:

	x_1	x_2
e_1	0,25	0,75
e_2	0,5	0,5

(Por exemplo, a probabilidade de obter x_1 com e_1 é 0,25.)

Admita que o delegante é neutro ao risco e que o agente é avesso ao risco. A função de utilidade do delegante é: $v = x - w$. A função de utilidade do agente é dada por: $u(w,e) = \sqrt{w} - g(e)$ com $g(e_1) = 40$ e $g(e_2) = 20$. A utilidade de reserva do agente é $\overline{U} = 120.$

a) Admitindo que a informação é simétrica (isto é, o esforço é observável), determine o contrato óptimo para cada nível de esforço. Qual é o nível de esforço óptimo para o delegante? (1.5 valores)

b) Admita agora que o esforço não é observável (ao contrário de x, que é verificável). Determine o contrato óptimo para induzir cada um dos níveis de esforço. Qual é o nível de esforço óptimo para o delegante na presença de risco moral? (2.5 valores)

c) Como é que a existência de risco moral influenciou o contrato óptimo? Neste exemplo, qual é o custo social associado ao problema de risco moral? (1 valor)